**ChromaStar lab 3: Main Sequence *M-R* and *T*eff*-R* relations**

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**Level:** Second year University

**Purpose:** To experimentally approximate the main sequence (MS) effective temperature – mass (*T*eff*-M*) and effective temperature – radius (*T*eff*-R*) relations.

*Optionally:* To infer the MS mass – mean density (*M-<ρ>*) and mass – luminosity (*M*-*L*bol) relations, and to compare the inferred *M*-*L*bol relation to that of a simple formula.

**Background:** The five basic global parameters of stars are the bolometric luminosity, *L*bol, the effective surface temperature, *T*eff, the radius, *R*, the mass, *M*, and the surface gravity, *g*. They are related by the two basic relations for spherical gravitating blackbodies: The Stefan-Boltzmann law of blackbody radiation, *L*bol = *σT*eff*44πR2*, and the definition of surface gravity, *g = GM/4πR2* in MKS units, where *L*bol is in erg s-1 , *T*effis in K, *R* is in cm, *M* is in g, *g* is in cm s-2, *σ* is the Stefan-Boltzmann constant and *G* is the gravitational constant, or *L*bol = *T*eff*4R2*, and *g = MR-2* in solar units (*L*Sun, *T*eff, Sun, *R*Sun, *M*Sun, *g*Sun). The *theorist’s* definition of the Hertzsprung-Russell diagram (HRD) should have logarithmic units (log10 *L*Sun *vs* log10 *T*eff) to be comparable to the *observer’s* HRD (*M*bol *vs* *B-V*), and taking the log10 of the two relations in solar units gives

log10 *L*bol *= 4* log10 *T*eff *+ 2* log10 *R* or log10 *R = -2* log10 *T*eff *+ 0.5*log10 *L*bol(1)

and

log10 *g* *=* log10 *M - 2* log10 *R* or log10 *R =* 0.5log10 *M – 0.5*log10 *g* (2)

Note that for fixed value of *R* equation (1) defines a straight *line of constant radius* in the theorist’s HRD with slope, *m=4* and *y*-intercept, *b=2* log10 *R* (note that in these units the *origin*, *O*, is at the Sun’s position). These lines of constant radius are displayed on the HRD in ChromaStar’s output and note that the line of *R=1* RSun crosses the MS at the Sun’s position.

**Apparatus:**

The ChromaStar stellar atmospheric modelling WWW application: ([www.ap.smu.ca/OpenStars/](http://www.ap.smu.ca/~ishort/OpenStars/GrayStar3/GrayStarV4.html) )

A spreadsheet application: (OpenOffice Calc (free!), MS Excel, …). You must be able to ‘Save’, ‘Export’, or ‘Print’ the file in a platform-independent format such as PDF – you might have to submit it electronically.

**Initial set-up:**

Make sure you are starting with a fresh ‘reload’ of ChromaStar so that all the input parameters have their default values (among other things, the stellar parameters will default to solar values - if you think that some values are not reverting to default, try clearing your browser’s history with all optional data types checked, and ‘reload’ again).

*Note* the position of the default model (the Sun) in the HRD – it should be centered on the MS where it crosses the line of constant radius for *R=1* RSun (after all, the Sun is an MS star and has *R=1* RSun by definition of RSun!).

In a spreadsheet application open a new document and save it with the filename “YourLastName-TeffMassLab”. At the top of the sheet, enter a meaningful *title*, the *date*, and your *name*, and the *course name*. You *might* have to submit the spreadsheet electronically.

**Procedure:**

1. In your spreadsheet, below the header information you have already entered, *log* the two stellar parameters from the “stellar” input panel that will remain *fixed* – log *g* and metal content (*[A/H]*) (they should be the default solar values!). This involves logging both the *name* of the parameter as it appears in the ChromaStar panel, and the corresponding *value*. Note that you will be *varying* *T*eff and mass - they should *not* be logged in the header.
2. In your spreadsheet, below the fixed parameters you have logged, leave several blank rows and establish a data table that will have eight columns. Give columns 1 and 2 the headings “Teff (K)” and “log(Teff)”, columns 3 and 4 the headings “M (MSun)” and “log(M)”, columns 5 and 6 the headings “R (RSun)” and “log(R)”, and column 7 and 8 the headings “Lbol (LSun)” and “log(Lbol)”.
3. Program columns 2, 4, 6, and 8 with a simple formula to take the log of the corresponding quantities in columns 1, 3, 5, and 7, respectively. You may need to access ‘help’ on how to program a formula with your spreadsheet application. (You should only have to program the formula *once* for the first row, then the remaining rows can be filled in by “selecting” them all and “pasting” the formula – that’s the power of spreadsheets!)
4. In the “stellar” input panel, find the control for the *effective temperature*, *T*eff, and change the value from the default (5778 K) to 3600 K (use the central text box to set the value precisely). Click the “Model” button and model your first test star. In the graphical output section, check the HRD – at this *T*eff value you should see that the star is no longer centered on the MS, but is now slightly above it (it *should* still be on the line of *R=1* RSun at this point). Experimentally adjust the star’s mass, *M*, and re-run the model until the star is re-centered on the MS (*ie*. *fit* the model to the MS). Note that this will necessarily mean the star is no longer on the line of *R=1* RSun – the new radius, *R*, and bolometric luminosity *L*bol –are displayed at the left in the output panel just above the HRD. Centering the star on the MS is judged by eye, and is necessarily an approximate procedure – it may be helpful to expand the HRD by zooming. In your data table, log this value of *T*eff (in K), the value of *M* (in MSun) that best centers the star on the MS, and the corresponding values of *R* (in RSun) and *L*bol (in RSun) in columns 1, 3, 5, and 7, and have the spreadsheet calculate and record the corresponding log values in columns 2, 4, 6, and 8.
5. Repeat step 3) for a range of input *T*eff values from 3600 to 9600 K (late-K to late-B class stars) with a sampling, *ΔT*eff, of 1000 K, then from 10000 to 40000 K with a *ΔT*eff value of 5000 K. This will lead to a substantial data table with many rows.
6. Have the spreadsheet application make a “line plot” or a “scatter plot” (*ie.* symbols with no connecting lines) of log *R (y*-axis) *vs* log *T*eff *(x*-axis) (*ie*. the logarithmic MS *T*eff *– R* relation). (This will involve “selecting” the entire data table and then opening the “Data” tab in the spreadsheet menus.) Note that the default plot type is probably something inappropriate (like “bar chart”) and you will have to choose something that looks like a line plot or a scatter plot. Make sure the plot is big enough for you to mark on it when doing the analysis below. This will be a bit tricky – the spreadsheet will want to plot up *all* the columns you selected *vs* log *T*eff. You will have to edit the plot so as to remove the unwanted relations – they are not useful, will distort the *y*-axis plot scaling, and are distracting clutter. Give the plot a meaningful *title*, and the axes the correct *labels*.
7. Repeat Step 6), with a separate graph, for log *R (y*-axis) *vs* log *M* *(x*-axis) (*ie*. the logarithmic MS *M* *– R* relation).
8. For the two end points *only*, *T*eff values of 3600 and 40000 K,estimate the uncertainties, *ΔM* (in MSun) and *ΔR* (in RSun) in the values of *M* and *R* that fit the MS. This can be judged by determining how much you can nudge the Mass dial up or down around the best fit *M* value and still convince yourself that the model is approximately centered on the MS.

From differential calculus, we know that *δ*log *x =* δ*x/x* – use this relation to evaluate the corresponding values of *Δ*log *M* and *Δ*log *R*. Record these uncertainties in your spreadsheet below the main data table.

1. **Optional:** In reality, the value of log *g* *also* changes along the MS, *decreasing* with increasing *T*eff from a value of about 4.6 at 3600 K to a value of 3.8 at 40000 K. For these two *T*eff values *only*, re-fit the model to the MS and record the best fitting log *M* and log *R* values in the spreadsheet.
2. **Required:** Print out your spreadsheet with the graph. You will need to mark on it in the analysis below, and hand it in with your submission.

**Analysis & Discussion:**

1. The log *R* *vs* log *M* relation for MS stars, *as modeled in the first 5 steps* of the procedure with *constant* log *g*, can be fitted with a straight line of the form *y = mx + b*. You may need to print out the plot and draw a straight line that you judge by eye to be the best fit through the data points. Find the slope, *m*, and the *y* intercept, *b*, of your MS log *R* *–* log *M* relation and estimate the uncertainties, *Δm*, and *Δb*, graphically – the vertical error bars, *Δ*log *R*, and horizontal error bars, *Δ*log *M*,estimated in Step 8 of the *Procedure* will help establish *Δm* and *Δb*.
2. From the second form of Eq. 2, we expect the slope, *m*accepted, of the log *R* *vs* log *M* plot for *constant* log *g* modeling to be 0.5. Does your graphically derived value of *m* agree with the value of *m*accepted, to within the uncertainty, *Δm*? (You may need to dust off your 1st year physics lab skills when it comes to the meaning of “agreement”!). *Note* that you *will* ***not*** *lose marks* if your result does not agree with the accepted one, but you *will lose marks* if your statement is not supported by the values of the accepted *m*, your derived *m*, and your *Δm*!
3. From the second form of Eq. 2, we expect the *y*-intercept, *b*accepted, of the log *R* *vs* log *M* plot to be *0.5*log10 *g*, where log10 *g* is the default value used up to Step 5. Does your graphically derived value of *b* agree with the value of *b*accepted, to within the uncertainty *Δb*?
4. log *R vs* log *T*eff plot: Pick three data points that are widely separated in log *T*eff value, and use the fitted log *R* value from Step 5 of the *Procedure* to *calculate* the value of log *L*bol from Eq. 1. Use your estimate of *Δ*log *R* from Step 8 of the *Procedure* to estimate *Δ*log *L*bol. For each of these three cases, do your log *L*bol values calculated from fitting models to the MS agree with the log *L*bol values displayed by ChromaStar, as recorded in your data table in step 5 of the *Procedure*, to within the uncertainty?
5. Optional: From Step 9 of the *Procedure* comment on *systematic errors* in the derived values of *M* and *R* from Step 5. How does assuming constant log *g* affect the derived values of *M* and *R* as a function of *T*eff?
6. **Optional:** Assuming stars are spherical, have your spreadsheet compute the volume, *V*, of MS stars of each *T*­eff. Then, using your *M* values from earlier, have it compute the average mass density, *<ρ>*, for each *T*­eff. The mass, *M*, MS stars increases with increasing *T*­eff value – how does the are more massive – how does the value of *<ρ>* vary with increasing *M* value? What is he explanation?
7. **Optional:** If you have also recorded the bolometric (*ie.* total) luminosity, *L*bol, during the *Procedure*, have the spreadsheet graph up your inferred logarithmic MS mass-luminosity (log *M-* log *L*) relation log *L*bol (*y-*axis) *vs* log *M* (*x-*axis) where *M* and *L*bol are in *solar units*. A simple approximation for the MS *M-L* relation is log *L*bol *= 3.5 log M*, (meaning that is *L*bol *= M3.5*), where *M* and *L*bol must be in *solar units* (*not* cgs or mks units!). Have your spreadsheet add the log *L*bol *= 3.5 log M* line – how does it compare to your inferred log *M-* log *L* relation.